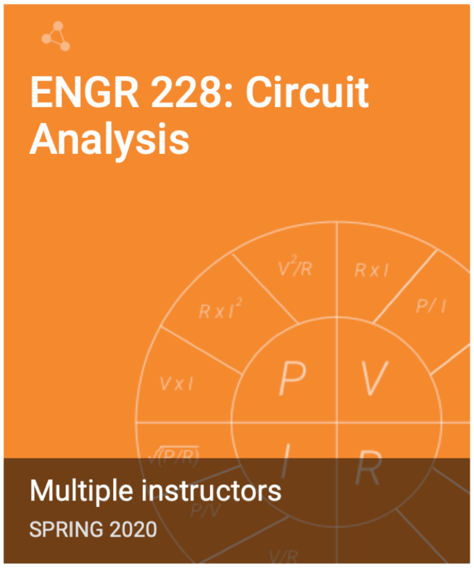


Chapter 5.5

Response of the RC circuit

Part I

Engr228 - Circuit Analysis

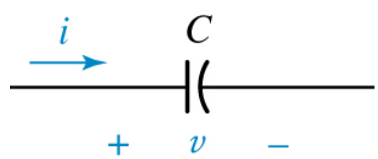
Spring 2020

Dr Curtis Nelson

Section 5.5 Objective

- Learn to:
  - Analyze the transient responses of RC circuits.

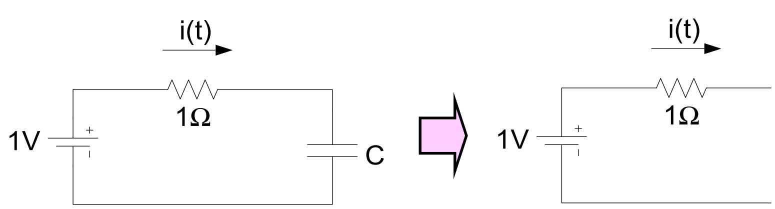
Capacitors



- The unit of capacitance is the Farad (F);
- 1 F = 1 Amp-Second/Volt = 1 Coulomb/Volt;
- The governing voltage and current relationship is:

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

DC Characteristics of a Capacitor

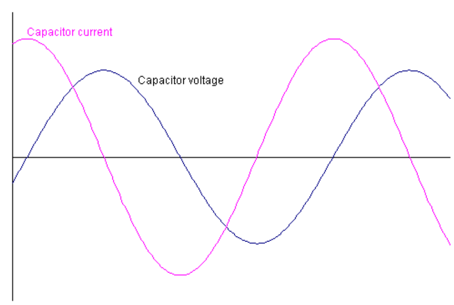


The capacitor acts like an "open circuit" at DC because the time rate of change of voltage is zero so, no current can flow through it.

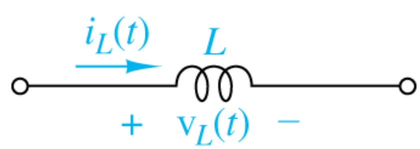
$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Voltage-Current Relationship in a Capacitor

Current and voltage in a capacitor are not in phase with each other. **For sinusoidal waves, the voltage across a capacitor lags the current through it by 90º.** (In other words, the current leads the voltage by 90º.) In the diagram below, the tall purple waveform represents the current through a capacitor and the shorter blue waveform represents the voltage across a capacitor.



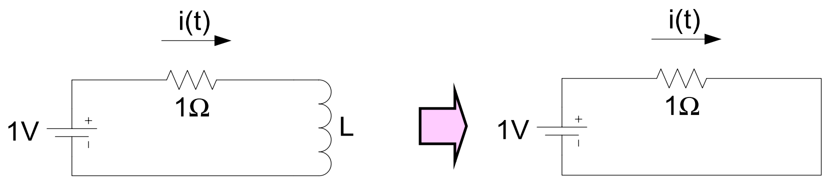
Inductors



- The inductor is often called a coil because physically coiling a wire greatly increases its inductance, especially if it is coiled around a magnetic material.
- The governing voltage and current relationship is:

$$v_L(t) = L\frac{di(t)}{dt}$$

DC Characteristics of Inductors

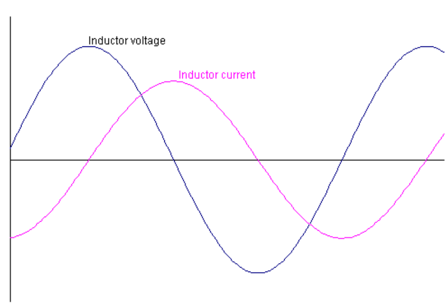


The inductor acts like a "short circuit" at DC because the time rate of current change is equal to zero.

$$v_L(t) = L\frac{di(t)}{dt}$$

Voltage-Current Relationship in an Inductor

Current and voltage in an inductor are not in phase with each other. **For sinusoidal waves, the voltage across an inductor leads the current through it by 90º.** (In other words, the current lags the voltage by 90º.) In the diagram below, the tall blue waveform represents the voltage across an inductor and the shorter purple waveform represents the current through the inductor.

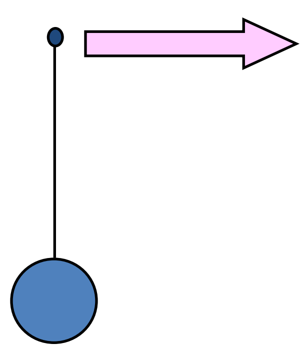


Types of First-Order Responses

- Circuits with one storage device (one capacitor or one inductor), are called **first-order circuits**.
- Their response to source excitations is composed of two parts:
  - Transient response, natural response, homogeneous solution (temporary position change)
    - Fades to zero over time.
  - Forced response, steady-state response, particular solution (permanent position change)
    - Follows the input;
    - Independent of time passed.

Mechanical Analogue

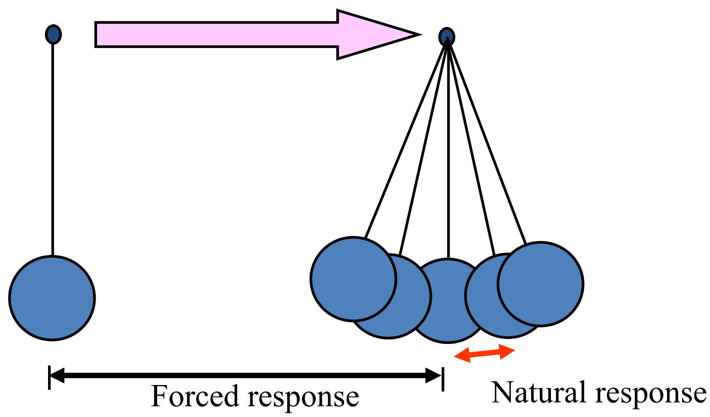
I am holding a ball with a rope attached. What is the movement of the ball if I move my hand to another point?



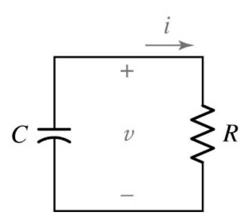
Two movements:

1. Oscillation
2. Forced position change

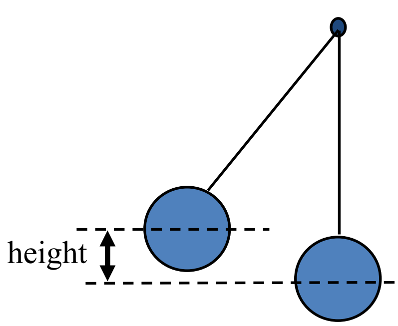
Mechanical Analogue



Source-Free RC Circuits



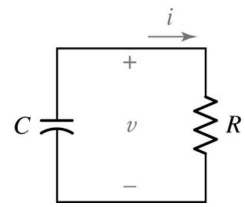
Capacitor C has energy stored so initial voltage is V_0 or V(0-)



Similar to a pendulum that is at a height h where the potential energy is nonzero.

Source-Free RC Circuits

- The derivation for the capacitor voltage is a node equation. To be consistent with the direction of assigned voltage:



$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 0$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0$$

- There are 2 ways to solve this first-order equation:
  - Assume the form of a solution.
  - Direct integration.

Solving Source Free RC Circuits

Assume the solution is of the form $v(t) = Ae^{st}$
where A and s are the constants that need to be solved for.

Substitute $v(t) = Ae^{st}$ into the equation: $\frac{dv(t)}{dt} + \frac{v(t)}{RC} = 0$

$$v_C(t) = V_{C0}e^{-\frac{t}{RC}} = V_{C0}e^{-\frac{t}{\tau}}$$

Time Constant

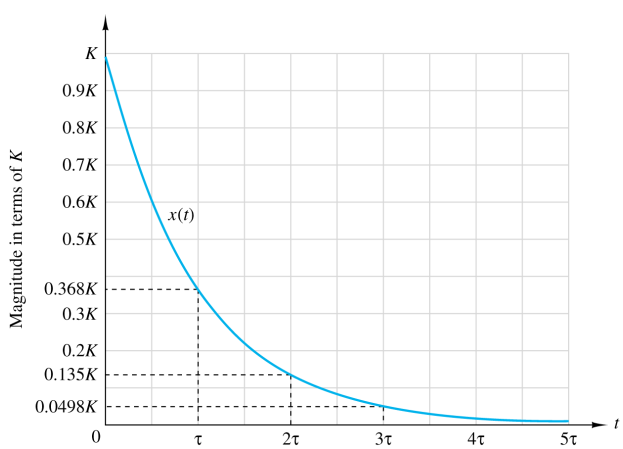
The term *RC* is called the **time constant** and is denoted by the symbol τ (*tau*).

$$\tau_C = RC \qquad \text{Units: seconds}$$

One time constant is defined as the amount of time required for the output to go from its initial value V(0) to 36.8% of its initial value.

$$e^{-1} = 0.368$$

Time Constant Graph

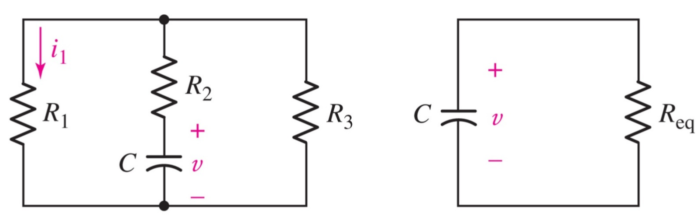


1^{st} Order Response Observations

- The voltage across a capacitor is the same *prior to* and *after* a switch at $t = 0$ seconds because this quantity cannot change instantaneously.
- Resistor voltage (or current) prior to the switch $v(0^-)$ can be different from the voltage (or current) after the switch $v(0^+)$.
- ***All*** voltages and ***all*** currents in an RC circuit follow the same natural response $e^{-t/\tau}$.

General RC Circuits

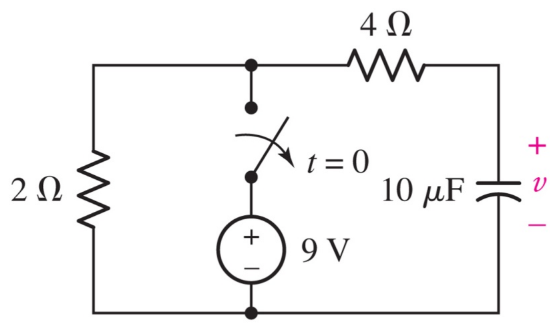
The time constant of a single-capacitor circuit will be $\tau = R_{eq}C$ where R_{eq} is the resistance seen by the capacitor.



Example: $R_{eq}=R_2+R_1R_3/(R_1+R_3)$

The Source Free RC Circuit

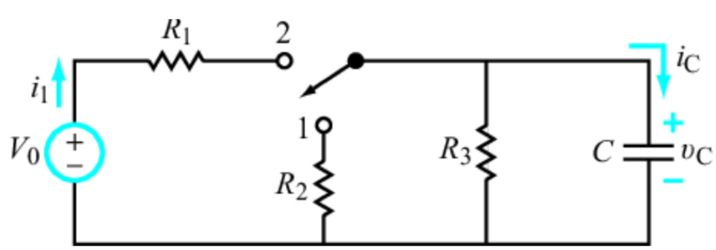
Assume the switch is in the closed position for a long time for t < 0. Find the voltage *v(t)* at t = 200µS.



v(t) = 321mV at t = 200µS

Example 5.5.2 Zybooks

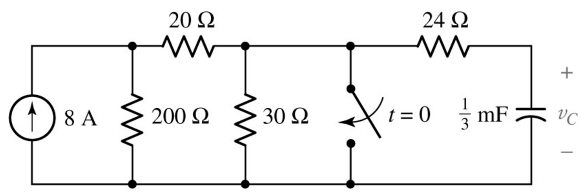
After having been in position 2 for a long time, the switch is moved to position 1 at t = 0. Given that V_0=12 V, R_1=30 kΩ, R_2=120 kΩ, R_3=60 kΩ, and C=100µF, determine v_C(t) for all time.



$v_c(t) = 8e^{-.25t}$

Example Problem

(a) Find $v_C(t)$ for all time in the circuit below.
(b) At what time is $v_C = 0.1v_C(0)$?



$v_C(0) = 192V$ $v_C(t) = v_C(0)e^{-125t}$

$v_C = 0.1v_C(0)$ when $t = 18.42mS$

Section 5.5 Summary

- You learned to:
  - Analyze the transient responses of RC circuits.